PHILOSOPHICAL TRANSACTIONS OF THE ROYAL SOCIETY A
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Re-Determination of the Mass of a Cubic Inch of Distilled Water

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VII. *Re-determination of the Mass of a Cubic Inch of Distilled Water.*

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THE evaluation of the mass of a cubic inch of distilled water, as at present accepted, was based on weighings made in 1798 by Sir G. SHUCKBURGH ('Phil. Trans.,' 1798, p. 133); and on measurements made in 1821 by Captain KATER ('Phil. Trans.,' 1821, pp. 316 and 326). Subsequent researches however, particularly those in relation to the mass of a cubic decimetre, show that it is desirable to re-determine the mass of the cubic inch of distilled water.

The result of SHUCKBURGH's experiments was that the cubic inch of distilled water at the temperature of 66° Fahr., the barometer being at 29·74 inches, weighed 252·422 grains, and this value as corrected by Captain KATER, became 252·458 grains at the temperature of 62° Fahr., the barometer being at 30 inches; or *in vacuo* ($t.$ = 62° Fahr.) the cubic inch of distilled water weighed 252·724 grains. This corrected value, 252·458 grains, has been adopted in various legislative enactments;—for instance the Weights and Measures Act, 5 Geo. IV., c. 74, section 5 (1824), declared that "a cubic inch of distilled water, weighed in air by brass weights, at the temperature of sixty-two degrees of FAHRENHEIT's thermometer, the barometer being at thirty inches, is equal to two hundred and fifty-two grains and four hundred and fifty-eight thousandth parts of a grain, of which the imperial standard Troy pound contains five thousand seven hundred and sixty."

Owing to doubt as to the true mass of a cubic inch of distilled water, the above section of the Act of 1824 was repealed in 1878, and has not been re-enacted. No re-determination of the mass of a cubic inch of distilled water in terms of the present imperial pound has yet been made; the above value, 252·458 grains, being based on the old Troy pound of 1758. Dr. WILD's investigations* appear indeed to show that later results obtained in the evaluation of a definite volume of distilled water, differ appreciably from those obtained in terms of the old Troy pound. If, for instance, the mass of a cubic decimetre of distilled water at 4° C., as originally determined by LEFÈVRE-GINEAU and TRALLÈS in 1799, is taken as 1000·000 grms.

* 'Bericht über die Arbeiten zur Reform der Schweizerischen Urmaasse,' Zürich, 1868.

Then the equivalent value of the cubic decimetre, as deduced from the SHUCKBURGH and KATER value of the cubic inch, would be	1000·480 grms.

Other investigations have given :—

(1825)	BERZELIUS, SVANBERG, AKERMANN	1000·296 ,,
(1834)	STAMPFER	999·653 ,,
(1841)	KUPFFER	999·989 ,,

The arithmetical mean of these five results is 1000·084 grms., and there is a difference of 0·827 grm. between the highest and lowest result. The difference between the results appears to arise mainly from inaccuracy in the measurement of temperature, and uncertainty as to the condition of the water used.

Dr. BROCH* revised a report by TRALLÈS on LEFÈVRE-GINEAU'S original work, and arrived at the conclusion that probably the true mass of a cubic decimetre of distilled water at its maximum density, and in a vacuum, is from 90 to 120 mgrms. less than 1000 grms.

Methods and Instruments.

It has been the object of the present investigation to ascertain the weight of water displaced by a body or gravimeter, whose masses, in air and *in vacuo*, and linear dimensions, had been carefully ascertained, rather than to determine the theoretical mass of a cubic inch of water. It has been considered that the mass of a definite volume of distilled water cannot well be ascertained by weighing the water contained in a vessel of a given capacity; or by the use of gravimeters, made only of one material (as brass), and of one particular form (as a sphere). Three gravimeters (or hydrometers) therefore, of the following forms were now adopted :—

Two gravimeters, C and Q, of cylindrical form, one hollow and the other solid, the third gravimeter S being hollow, and of spherical form. The hollow gun-metal circular cylinder C (fig. 1, page 340) was nearly nine inches in height and diameter, and was adopted as being a body the weight and dimensions of which might give the least probable error in the several operations of weighing in air and in water. The gravimeter C was protected form oxidation by platinising, it having been first made air-tight. For the purpose of linear measurement, there were traced on the gravimeter a series of lines, as indicated in fig. 1. These lines were cut rather deeper than was desirable, but the space taken up by them on the surface of the cylinder has been considered.

* 'Procès-Verbaux—Commission Internationale du Mètre. Réunions des Membres Français. 1873.'

The cylinder Q was adopted as a solid gravimeter because the density and expansion of quartz are well known, as well as on account of the hardness of quartz, its capability of receiving a high polish and absence of hygroscopic properties. Q is a pure crystal, and was originally adjusted under the directions of Dr. VOIT, its coefficient of linear expansion for 1° C. being taken as 0·00000781 in a direction parallel to the principal axis of the crystal.

The third gravimeter S, is a hollow brass sphere, having on its surface two engraved lines for the purpose of linear measurement, and its approximate diameter is six inches.

Water.

Expansion of Water.—As the weighings of the gravimeters in water could not be actually made at the normal temperature of 62° Fahr., corrections were necessary for the density of water at various temperatures. There has not yet been made any determination of the rate of expansion of water which might alone be accepted, and we have therefore to adopt the mean result of several selected determinations. ROSSETTI has stated the result of his experiments (' Atti del Istituto Veneto,' 1867–8), as well as those of KOPP (1847), PIERRE (1845, 1852), DESPRETZ (1839), HAGEN (1855), and MATTHIESSEN (1866), in a table which gives 1·001121 as the ratio of the density of water at 62° Fahr. (16°·6 C.) to its maximum density at 4° C. It does not appear whether ROSSETTI corrected the earlier results by more recent determinations of the rates of expansion of mercury and of glass; which corrections at some temperatures would affect the last place of decimals in the above expression (1·001121). The differences between the above determinations would also affect the present investigation to ± 0·0009 grain in the mass of the cubic inch of water.

In 1870 Dr. FOERSTER,* after a critical examination of the results above referred to, as well as of those of JOLLY (1864) and W. H. MILLER (1856),† adopted the mean results of MILLER and ROSSETTI (after SCHIAPARELLI, 1868); which give 1·001118 as the ratio at 62° Fahr.; or if the maximum density of water is taken at the temperature of 4° C., then at 62° Fahr. the density of water may be expressed by 0·998881; and these mean results have, therefore, been followed in the present investigation.

* ' Metronomische Beiträge,' No. 1, Berlin, 1870.

† ' Phil. Trans.,' 1857 (Part III. for 1856).

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TABLE I.—Density of Water.

Logarithms of Ratios of the Maximum Density of Water to its Density at *t*.

t.	Log. W_t.	Diff.	*t*.	Log. W_t.	Diff.
4° C.	0·0000000		17° C.	0·0005098	
5	36	36	18	5867	769
6	136	100	19	6682	815
7	299	163	20	7544	862
8	524	225	21	8451	907
9	808	284	22	9401	950
10	1152	344	23	10393	992
11	1553	401	24	11426	1033
12	2011	458	25	12498	1072
13	2523	512	26	13609	1111
14	3090	567	27	14757	1148
15	3708	618	28	15941	1184
16	4378	670	29	17158	1217
17	0·0005098	720	30	0·0018409	1251

Condition of the Water.—The water was twice distilled in pure tin and glass stills, and was found to be free from any impurities likely to affect the weighings. It was deprived of air by boiling, and no correction for the absorption of air was therefore made. It was, however, found that at the temperature of 62° Fahr. (B. = 30 inches) a cubic foot of distilled water, freed from air, weighed about 321 grains more than when nearly "saturated" with air.

Standards of Length and Comparators.

Standards of Length and Comparators.—For the purpose of measuring the external dimensions of the gavimeters in inches of the present imperial yard, four 9-inch end standards of length were used; as well as a steel 6-inch and a steel 3-inch standard (1884). At 62° Fahr., the true mean length of the four 9-inch standards was 8·99975 inches ± 0·00001 inch; the 6-inch and 3-inch standards having a true length of 6·00020 and 2·99975 inches respectively. For intervals between 0·1 and 0·01 inch there was also used a subdivided inch "D," the accuracy of the subdivisions of which had been measured by the Standards Commission in 1868 to ± 0·000005 inch.

For the comparison of the dimensions of the gravimeters with the standards of length, there were used two comparators of well-known forms; one a Whitworth contact comparator, the other a micrometer-microscope comparator designed by Mr. J. SIMMS. By means of the former comparator measurements by touch might be made to 0·0001 inch; and optically to 0·000025 inch, by means of the microscopes.

When two standards or measures of extension are to be compared by means of the Whitworth comparator, first one standard and then the other is placed in a horizontal position between two contact points; one of which is a fixed point, the other being a movable point or the termination of a micrometer-screw. Any difference, in parts of an inch, between the two standards can then be measured by means of the screw, the value in parts of an inch of one revolution or whole turn of the screw being known. In this comparator, the appreciation of the moment of contact is ascertained by means of a "gravity-piece." When the gravimeter was placed in position, one of its sides was in contact with the fixed point, its other side being in contact with the gravity-piece, which was interposed between the gravimeter and the screw, and the moment of contact between the gravimeter and the screw was then ascertained by gradually turning or releasing the screw until the gravity-piece fell by its own weight.

In the general form of the ordinary micrometer-microscope comparator, or optical beam-compass, there are two fixed microscopes, the distance between which is determined by comparison with a standard of length. The micrometer heads of the microscopes are divided into 100 divisions, the mean value of one division being equal to nearly 0·00003 inch; and the linear standards or measures of extension are compared by being alternately placed under the microscopes, a microscope being fixed over either end of the standard under observation.

As the comparators were only required for the purpose of measuring small differences between the standards and the gravimeters, it was unnecessary to verify the whole run of the micrometer screws; but the particular parts of the screws used were verified by comparison with the subdivided inch D.

There were no defining lines marked on the gravimeter Q by which its dimensions could be measured, although certain lines, as previously mentioned, were engraved on C and S, for the purpose of indicating generally the particular parts of the surface of the gravimeter brought under measurement. Therefore, in the microscopic comparisons of Q, it became necessary to develop defining lines, visible through the microscope, and this was done after the method of FIZEAU and CORNU.

When the gravimeter was placed in position under the fixed microscopes, two pencil points, made of polished silver, were brought nearly into contact with the gravimeter, within 0·001 inch of the gravimeter at each end. The "points" did not therefore actually touch the sides of the gravimeter, but were so reflected as to form an apparent line of contact, visible through the microscope, on either side of the gravimeter. The actual distance between the two apparent lines formed by the "points" and their reflections, as measured by the fixed microscopes, was then compared with a standard of length which was placed under the microscopes after the removal of the gravimeter.

Rates of Expansion of the Gravimeters by Heat.—The rates of expansion by heat of the three gravimeters were not ascertained by actual experiment, as the probable error which would arise in ascertaining the rate of expansion of a body having the particular form of either one of the gravimeters would be greater than the probable error of experiments on the dilatation of a body of the simple form of a rod or bar.

The gravimeter C, for instance, is made of a gun-metal alloy, the coefficient of linear expansion of which alloy was found by SHEEPSHANKS in 1848 to be 0·00000917 for 1° Fahr. It was subsequently found by CLARKE (1860) to be 0·00000986; FIZEAU, also, has shown that SHEEPSHANKS' coefficient was too low, owing possibly to the form of the mercurial thermometer adopted in 1848. For the gravimeter C, CLARKE'S coefficient of linear expansion has been therefore taken. For the gravimeter S the coefficient of linear expansion for 1° Fahr., 0·0000104, has been taken; and for Q the cubic expansion 0·00001924 for 1° Fahr.

Thermometers.

Six standard mercurial thermometers were used—

Centigrade	4,517
,, ...	4,518
,, ...	4,575
Fahrenheit	430
,, ...	12,765
,, ...	20,065

The errors of these thermometers were originally determined in relation to the hydrogen thermometer at the Bureau International des Poids et Mesures, Paris, and were re-determined after the experiments. Thermometers 4517 and 4518 were made of a hard glass (including 71·5 silica, 14·5 lime, 11·2 soda, 1·3 alumina, 0·7 sulphuric acid, and 0·3 potash), and were highly sensitive.

Thermometer No. 4517.—Divided into 0°·1 C. from 4°·3 to + 103°·4. Distance from middle of reservoir to 0° C. is 53 mm. Total length of thermometer is 702 mm. Length of a degree is 5·290 mm.

CORRECTION for Calibration.

Divisions.	Corrections.	Divisions.	Corrections.
°	°	°	°
0	0·0000	50	+ 0·1668
+ 2	0·0068	60	+ 0·1599
8	0·0393	62	+ 0·1520
16	+ 0·0651	70	+ 0·1163
18	+ 0·0639	80	− 0·0082
20	+ 0·0635	90	− 0·0257
30	+ 0·1039	100	± 0·0000
40	+ 0·1246		

Divisions equidistant. Probable error of correction ± 0°·0010 C.

Coefficient of pressure (or pressure on reservoir) for one millimetre of mercury, as applied when thermometer was used in a vertical position is 0·0001025.

Boiling-point in a horizontal position is 100°·0242 ± 0·000011. The value of a division in "normal" degrees being 0°·999758 ± 0°·000011.

Error of zero-point, when the thermometer was placed in a horizontal position, was 0°·074 C.

Thermometer No. 4518, is a Centigrade thermometer of similar form to 4517.

Boiling-point in a horizontal position is 100°·0665. The value of a division being 0°·999335 ± 0·000017.

Zero-point; horizontal position, − 0°·087.

Thermometer No. 4575.—Divided into 0°·1 C. from − 4°·1 to 55°·1. Distance from middle of reservoir to 0° C. is 64 mm. Total length of thermometer, 482 mm. Divisions not equidistant, but by comparison with a standard thermometer the following corrections were obtained:—

Scale.	Corrections.
°	°
5·1	+ 0·014
10·7	+ 0·007
15·2	+ 0·011
20·2	− 0·006
24·9	− 0·024
30·1	− 0·045
35·0	− 0·051

Coefficient of pressure is 0·0002264.

Zero-point; horizontal position, + 0°·314.

Thermometer; Kew, 430.—Divided into 1° Fahr. from − 10° to + 217° Fahr. Nos. 12,765 and 20,065. Divided into tenths of a degree from 20° to 84° Fahr.

	430.	12,765.	20,065.
	mm.	mm.	mm.
Distance in millimetres from middle of the reservoir to 32° Fahr.	228·3	134	80·3
Length of a degree	1·908	2·75	4·27
Total length of the thermometer	620	161	333

It was found that the construction of the above thermometers, Kew, 12,765, and 20,065, did not permit of exact calibration, and the following corrections were determined in a vertical position, by comparison therefore with a standard thermometer.

Kew No. 430.		No. 12,765.		No. 20,065.	
Scale readings.	Corrections.	Scale readings.	Corrections.	Scale readings.	Corrections.
° F.	° F.	° F.	° F.	° F.	° F.
41·5	+ 0·013	41·6	+ 0·121	41·7	− 0·001
50·2	+ 0·047	50·4	+ 0·045	50·4	+ 0·071
59·0	+ 0·060	59·3	− 0·003	59·2	+ 0·124
68·1	+ 0·101	68·5	− 0·037	68·4	+ 0·017
76·8	+ 0·013	77·0	+ 0·027	76·9	+ 0·104
86·1	+ 0·080	86·4	+ 0·065		
94·7	+ 0·130	95·0	+ 0·082		

Estimated probable error of correction is 0°·01 Fahr.

The coefficients of interior pressure at 100° C. for one millimetre of mercury is 0·0004334, for No. 430; and for the other thermometers the coefficients of pressure were stated as :—

12,765 . . . . . 0·0002950 ± 0·0000014.
20,065 . . . . . 0·0002039 ± 0·0000008.

Zero-points :—

Kew, 430 . . . . 32°·212 Fahr.
12,765 . . . . 32°·442 „
20,065 . . . . 32°·418 „

Weighings.

Weighings.—For the weighings three balances were used, by means of which differences of $\frac{1}{100}$th, $\frac{1}{2000}$th, and $\frac{1}{10000}$th of a grain might be ascertained respectively; for weighings in water it was possible, however, to weigh only to $\frac{1}{100}$th of a grain. The weighings were made by BORDA'S method of counterpoise, the position of equilibrium of the beam, and the weight of water displaced by the cylinder being calculated after the methods given by MILLER and BROCH.

The gravimeter was suspended in a glass vessel by a platinum wire hook, and was surrounded by at least two inches of water, the depth of the water being regulated so that it always rose to the same height in the vessel, whether the gravimeter was suspended therein or the wire hook only. The wire hook was kept polished, any water found to adhere to its upper surface being either wiped off or dried off by a blowpipe before the weighings were taken.

In such weighings in water minute bubbles of air are found to be carried down by the gravimeter or by the wire by which it is suspended, but by repeated immersions, and examination with a small telescope and a glow-lamp, it is possible to avoid appreciable error in this direction. When not in use the gravimeters were kept in air of similar temperature to that at which the weighings were made.

It is, of course, difficult, during such weighings, to find the precise temperature of a comparatively large volume of water, and as any uncertainty of 0°·2 Fahr. in the reading of the thermometer during the weighing of the cylinder C in water would amount to nearly three grains, it was desirable not only to use sensititive thermometers, but to place their bulbs at proper depths, and to read them quickly by means of cathetometers during the periods of weighing.

All weighings in air were reduced to "normal air," or air which at the temperature of 62° Fahr., the barometer being at 30 inches at 32° Fahr., reduced to latitude 45° and at sea level, contains four volumes of carbonic anhydride in every 10,000 volumes of air, and also contains two-thirds of the amount of aqueous vapour contained in saturated air. This is the average condition of the air at Westminster (latitude 51° 29′ 53″, at 16 feet above sea level), where the weighings were made. A litre of such air, if dry, but containing the above proportion of carbonic anhydride, would weigh 1·293934 grammes; and, if dry, but containing no carbonic anhydride, would weigh 1·293519 grammes.

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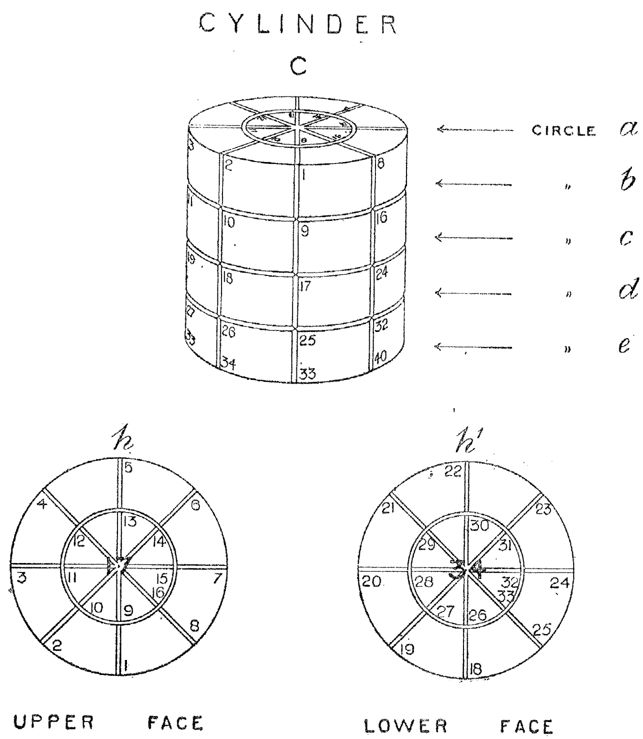
Details of Measurements and Weighings.

The following are details of the measurements and weighings of the three gravimeters C, S, and Q; together with statements of the results obtained.

Fig. 1.



HEIGHT of Cylinder C.

Date, 1888.	t. Fahr.	Height ($h - h'$, fig. 1). In divisions of the Whitworth micrometer. Outer circle. 1–18.	2–19.	3–20.	4–21.	5–22.	6–23.	7–24.	8–25.	Mean.	Mean reading of Whitworth micrometer for standard 9-inch bars.
	°	div.	div.	div.	div.	div.	div.	div.	div.	div.	div.
November 1	62·5	491·3	503·5	501·6	502·4	497·3	502·6	498·9	503·5	500·1	486·0
,, 5	62·5	497·6	501·9	501·4	501·8	501·3	502·7	502·4	502·2	501·4	484·5
,, 7	63·2	496·6	504·1	502·5	506·5	501·1	505·2	506·4	502·7	503·1	484·9
,, 8	62·0	485·7	503·2	502·2	502·2	500·3	501·9	499·1	502·1	499·6	483·8
,, 13	63·4	488·6	504·6	503·7	505·0	501·1	505·2	506·3	504·4	502·3	484·5
Mean =	62·72	491·96	503·46	502·28	503·58	500·22	503·52	502·62	502·98	501·3	484·74

In the measurement of C, the results are stated in divisions of the Whitworth micrometer, one division of which is equal to 0·0001 inch. Each result is the mean of three readings of the micrometer.

Date. 1888.	t. Fahr.	Height ($h - h'$, fig. 1). Inner circle. 9–26.	10–27.	11–28.	12–29.	13–30.	14–31.	15–32.	16–33.	Mean.	Mean reading of Whitworth micrometer for standard 9-inch bars.
	°	div.	div.	div.	div.	div.	div.	div.	div.	div.	
November 1	63·0	505·0	504·4	505·1	511·4	509·2	518·5	514·2	515·6	510·4	486·0
,, 5	63·5	501·2	502·0	498·5	511·3	510·9	514·3	508·6	509·7	507·0	484·5
,, 7	62·0	505·6	507·6	502·8	513·1	512·3	514·1	513·6	512·4	510·2	484·9
,, 8	62·1	501·2	503·6	499·8	508·6	508·6	511·4	508·1	509·0	506·3	483·8
,, 13	63·4	505·7	507·7	505·2	513·4	512·3	514·2	513·7	512·5	510·6	484·5
Mean =	62·8	503·74	505·06	502·28	511·56	510·66	514·5	511·64	511·84	508·9	484·74

Height ($h - h'$, fig. 1). Centre.			
Date. 1888.	t. Fahr.	17–34.	Mean reading of Whitworth micrometer for standard 9-inch bars.
	°	div.	div.
November 1	62·5	509·5*	486 0
" 5	63·2	506·7	484·5
" 7	61·5	512·1	484·9
" 8	62·1	506·9	483·8
" 13	63·0	506·5	484·5
Mean =	62·46	508·34	484·74

At 62° Fahr. the mean height of the cylinder C was therefore taken at 9·00202 inches.

* Ten consecutive micrometer readings on November 1 ($t = 62·5°$), gave a result by one observer as follows :—

509·8 div.
·7
·0
·5
·2
·6
·6
·6
·5
509·4

509·5 Mean.

DIAMETER of Cylinder C.

Date. 1888.	t. Fahr.	Circle a.—Number of diameter measured.									Mean reading of Whitworth micrometer for standard 9-inch bars.
		1–5.	2–6.	3–7.	4–8.	5–1.	6–2.	7–3.	8–4.	Mean.	
	°	div.	div.	div.	div.	div.	div.	div.	div.	div.	div.
November 1	64·5	449·7	500·6	497·5	498·3	498·8	501·2	500·5	497·9	499·1	486·3
,, 3	62·8	501·2	501·2	495·6	501·2	496·7	495·7	496·1	501·7	498·6	484·5
,, 6	61·5	501·5	500·8	501·0	501·5	502·3	501·4	501·7	501·8	501·5	484·4
,, 9	62·3	501·3	500·7	501·1	501·2	502·0	501·4	501·8	502·2	501·5	484·1
,, 12	62·4	493·2	491·8	491·8	492·3	492·6	492·6	492·6	493·2	492·5	483·2
Mean =	62·70	499·36	499·02	497·4	498·9	498·48	498·46	498·54	499·36	497·46	484·5

The measurements from (1–5) to (4–8), for instance, were taken with the cylinder C resting on its lower face. The position of the cylinder was then reversed, so that it rested on its upper face, in which latter position the measurements from (5–1) to (8–4) were then taken, the relative positions of the points on the cylinder brought into contact with the Whitworth micrometer-screw being thus reversed. Although, therefore, "1–5" and "5–1" are apparently the same straight line or diameter, yet the points between which the line would fall would be slightly different in each case.

Date. 1888.	t. Fahr.	Circle b.—Number of the diameter measured.									Mean reading of Whitworth micrometer for standard 9-inch bars.
		9–13.	10–14.	11–15.	12–16.	13–9.	14–10.	15–11.	16–12.	Mean.	
	°	div.	div.	div.	div.	div.	div.	div.	div.	div.	div.
November 1	64·1	501·5	495·9	495·5	494·0	495·4	493·7	492·8	494·5	495·9	486·3
,, 3	64·0	496·3	494·4	494·6	496·6	498·0	492·7	493·0	501·5	495·9	484·5
,, 6	61·7	492·7	490·8	490·5	491·0	490·8	491·3	491·5	492·3	491·4	484·4
,, 9	62·0	492·5	490·7	490·5	491·2	491·0	491·3	491·6	492·5	491·4	484·1
,, 12	62·4	492·4	492·0	492·1	491·8	492·6	492·5	491·7	492·5	492·2	483·2
Mean =	62·84	495·08	492·76	492·64	492·92	493·56	492·3	492·12	494·66	493·26	484·5

Date. 1888.	t. Fahr.	Circle c.—Number of the Diameter Measured.									Mean reading of Whitworth micrometer for standard 9-inch bars.
		17–21.	18–22.	19–23.	20–24.	21–17.	22–18.	23–19.	24–20.	Mean.	
	°	div.	div.	div.	div.	div.	div.	div.	div.	div.	div.
November 1	64·0	492·8	494·3	492·3	492 2	494·8	491·4	493·6	492·0	492·9	486·3
,, 3	63·2	494·7	492·0	493·7	493·4	494·9	494·2	495·6	494·6	494·1	484·5
,, 6	61·9	492·2	491·7	493·2	492·1	492·4	491·1	493·8	493·4	492·5	484·4
,, 9	62·2	492·3	491·7	493·0	492·1	492·3	491·0	493·6	493·2	492·4	484·1
,, 12	64·0	494·0	493·8	494·4	493·5	494·5	494·5	494·6	494·1	494·2	483·2
Mean =	63·06	493·2	492·7	493·32	492·66	493·78	492·44	494·24	493·46	493·22	484·5

Date. 1888.	t. Fahr.	Circle d.—Number of the Diameter Measured.									Mean reading of Whitworth micrometer for standard 9-inch bars.
		25–19.	26–30.	27–31.	28–32.	29–25.	30–26.	31–27.	32–28.	Mean.	
	°	div.	div.	div.	div.	div.	div.	div.	div.	div.	div.
November 1	62·8	498·2	497·3	498·0	498·1	497·5	494·2	498·9	498·5	497·6	486·3
,, 3	62·3	487·7	488·8	489·0	491·5	488·9	488·3	491·6	492·6	489·8	484·5
,, 6	62·1	487·7	488·6	489·2	491·4	489·1	488·3	488·6	487·3	488·8	484·4
,, 9	62·3	487·6	488·4	489·1	491·4	489·1	488·2	488·6	487·4	488·7	484·1
,, 12	64·0	491·6	491·6	491·5	492·3	490·7	492·3	491·9	491·5	491·7	483·2
Mean =	62·7	490·56	490·94	491·36	492·94	491·06	490·26	491·92	491·46	491·32	484·5

Date. 1888.	t. Fahr.	Circle e.—Number of the Diameter Measured.									Mean reading of Whitworth micrometer for standard 9-inch bars.
		33–37.	34–38.	35–39.	36–40.	33–37.	34–38.	35–39.	36–40.	Mean.	
	°	div.	div.	div.	div.	div.	div.	div.	div.	div.	div.
November 1	62·5	507·8	507·8	510·3	511·2	511·1	508·1	512·0	511·6	509·9	486·3
,, 3	64·0	505·6	502·0	510·0	507·5	509·0	505·6	504·4	506·7	506·4	484·5
,, 6	64·0	500·6	498·7	502·7	502·7	502·6	502·4	505·0	503·6	502·3	484·4
,, 9	62·3	500·6	498·6	502·7	502·7	502·5	502·4	505·0	503·6	502·3	484·1
,, 12	67·5	503·4	502·3	506·4	506 4	506·3	505·5	508·7	506·7	505·7	483·2
Mean =	64·06	503·6	501·88	506·42	506·5	506·1	504·8	507·02	506·44	505·32	484·5

The lines traced on C have the effect of diminishing its cubic capacity; these lines are V-shaped, and have a depth of nearly 0·018-inch, with an average breadth at the top of 0·01-inch. There are 24 vertical lines, each nearly 9 inches long; three circular lines, each nearly 28 inches long; besides two circular lines nearly 14 inches long. The edges also of the cylinder were not quite sharp.

If, therefore, the diameter of C is taken at 9·00115 inches, then the cubic contents of C may be taken at 572·80365 cubic inches.

WEIGHING of Cylinder C in Air.

Date, 1888.	Description of Weights used (w).	Result.	Mean t. Fahr.	Barometer. Inches.
	Before immersion.		°	in.
Nov. 16 . .	14, 7, 4, 1 lb., R;* 2, 1 oz.; 8 dr.; 100, 30, 10, 5 grains	C ≏ w + 0·086 grain . . Or C = 183676·25 + 0·086 grains	57·5	29·98
„ 19 . .	14, 7, 4, 1 lb., R;* 2, 1 oz.; 8 dr.; 100, 30, 10, 5 grains	C ≏ w + 0·009 grain . . C = 183676·25 + 0·009 grains	57·3	29·98
	After immersion.			
„ 22 . .	14 lb. . . . . 8 dr.; 100, 30, 10, 5 grains	C ≏ w + 0·998 grain . . Or C = 183675·25 + 0·998 grains.	60·1	30·22
„ 29 . .	14 lb. . . . . 8 dr.; 100, 30, 10, 5 grains	C ≏ w + 1·052 grain . . C = 183675·25 + 1·052 grains	58·2	29·34

The following are the weights of air displaced by C and w respectively.

The density of the weights used (w) was 8·0298, but their values were always corrected to the density of 8·1430, or to the density (Δ) of the brass representative of the Imperial Standard Pound adopted in this country.

	C displaces	w displaces
1888, Nov. 16 . .	177·03 grains . .	28·20 grains.
„ „ 19 . .	177·63 „ . .	28·10 „
„ „ 22 . .	178·66 „ . .	28·27 „
„ „ 29 . .	173·50 „ . .	27·45 „

In normal air, C displaces 176·08, and w 27·47 grains, whence the weight, in grains, of the cylinder was taken as follows:—

In vacuo.	In normal air.
C = 183797·198 grains . . .	183676·066 grains.

* R, Reference Avoirdupois Standards; having small errors in relation to the present Imperial Pound, which errors were allowed for.

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As the weighings of the gravimeters were made against standard weights, which were parts and multiples of the Imperial Standard Pound, it became necessary, in order to find the true masses of the gravimeters, to reduce the weighings to a vacuum, as the Imperial Standard Pound itself has its true mass stated *in vacuo*.

WEIGHING of C in water.

Date, 1888.	Weight of counterpoise (w^{-1}).	Mean t. water.	Mean t. air.	B.
	grains.	°	°	in.
Nov. 20	39091·84	58·87	56·0	30·14
„ 21	39109·85	58·10	57·2	30·15
„ 22	39109·87	56·38	56·5	30·22
„ 24	39141·86	61·42	59·5	30·30
„ 27	39090·50	57·61	56·8	29·38
„ 29	39125·00	58·00	58·2	29·34
	39111·49	58·40	57·37	29·92

The differences in the above weighings of C on the several days were considered to be owing partly to thermometric variations, and partly to the presence of minute bubbles of air carried down by this large cylindrical body. Results, however, could not be rejected merely because they did not always closely agree.

At 32° Fahr., to which temperature densities are reduced :

$$\Delta C_{32^\circ} = \begin{array}{l} 1{\cdot}27029 \\ 1{\cdot}27050 \\ 1{\cdot}27061 \\ 1{\cdot}27055 \\ 1{\cdot}27036 \\ 1{\cdot}27064 \\ \hline 1{\cdot}27049 \end{array}$$

or log $\Delta C_{32^\circ} = 0{\cdot}1040169$.

The logarithm of the volume of C in grains at 62° Fahr., may be expressed as follows :—

$$\text{Log vol. C} = \begin{array}{r} 5{\cdot}1599338 \\ 8626 \\ 8272 \\ 8476 \\ 9102 \\ 5{\cdot}1598148 \end{array}$$

or vC = 144499·48 grains weight of water at 62° Fahr.

As the cubic contents of C = 572·80365 cubic inches, the mass of one cubic inch of distilled water (Δ 0·998881) at 62° Fahr., B. = 30 inches, has been taken as 252·267 grains.

Diameter and Height of Q.

The diameter of Q was measured at 25 different points, each measurement being the mean of three readings. The method of Fizeau was followed, the true distance between the "points" ($p - p'$) being ascertained after the diameter of Q had been measured.

1889.	Mean readings in inches in terms of $p - p'$.	t.
	inches.	°
(1) Jan. 8 . . .	3·083513	61·8
(2) ,, . . .	3·083546	
(3) ,, 9 . . .	3·083684	61·8
(4) ,, . . .	3·083792	
(5) ,, 10 . . .	3·083846	62·0
(6) ,, . . .	3·083921	
(7) ,, . . .	3·084083	61·9
(8) ,, . . .	3·084123	
(9) ,, . . .	3·084086	61·7
(10) ,, . . .	3·083924	
(11) ,, . . .	3·083910	
(12) ,, 11 . . .	3·083923	61·6
(13) ,, . . .	3·083964	
(14) ,, . . .	3·083982	
(15) ,, . . .	3·084013	
(16) ,, . . .	3·084214	61·5
(17) ,, . . .	3·084267	
(18) ,, . . .	3·084285	
(19) ,, . . .	3·084235	61·8
(20) ,, 15 . . .	3·084108	
(21) ,, . . .	3·084124	
(22) ,, . . .	3·084086	61·9
(23) ,, . . .	3·084093	
(24) ,, . . .	3·084004	
(25) ,, . . .	3·083902	

At t. 62° the true distance been $p - p'$ was found to be 3·08234 inches by the 3-inch steel measure (2·999749 inch). Hence at 62° Fahr. the diameter of Q was taken at 3·08399 inches.

HEIGHT of Q as measured at 25 different points by Whitworth Micrometer.

No.	1889.	Mean reading.	t.
		Divisions of micrometer.	°
1	Jan. 7	+4555	61·8
2	"	+4500	
3	"	+3998	
4	"	+3842	
5	"	+3459	
6	" 8	+4335	61·7
7	"	+4257	
8	"	+3980	
9	"	+3938	
10	"	+3935	
11	"	+4203	
12	"	+4200	
13	"	+4000	
14	"	+4000	
15	" 10	+4000	62·0
16	"	+4038	
17	"	+3925	
18	"	+3875	
19	"	+3928	
20	"	+4054	
21	" 12	+3695	61·8
22	"	+3700	
23	" 15	+4000	
24	"	+4358	
25	"	+4450	
		Mean +4049	

For the standard 3-inch measure, the mean reading was found to be = 4799·5 divisions, hence the height of Q was taken as 3·088485 inches.

By the above measurements of the height of Q it may be seen that the quartz gravimeter is not of true cylindrical form, and that its height could only be approximately ascertained. After allowing for bevels on the upper and lower edges of the cylinder, as measured by the microscopic comparator, its cubic contents were taken as 23·04015 cubic inches.

WEIGHING of Q in Air.

Date, 1888.	Description of weights used (w).	Result.	Mean t. Fahr.	Barometer inches.
Jan. 3, 10 A.M.	Gilt lbs. 31, 32, + gilt grains 1000, 300, 100, 20, 10, 1, 0·5	grains 15426·68	57°·35	30·81
	After immersion.			
" 5 P.M.	.. ..	15426·68	57·33	30·81

The following are the weights of air displaced by Q (Δ 2·6505) and w (Δ 8·4924) respectively :—

Q.	w.
7·3496 gr.	2·2944 gr.
7·3495	2·2943

In normal air Q displaces 7·0865 gr. and w displaces 2·2123 gr., whence, *in vacuo* and in normal air respectively, the weight in grains of the sphere would be :—

In vacuo.	In normal air.
Q = 15429·55515	15426·95495 gr.

WEIGHING of Q in Water.

Date, 1888.	Weight of counterpoise w'.	Mean t. water.	Mean t. air.	Barometer inches.
Jan. 3 . . .	Gilt lb. 32 + 4 oz. + 500, 300, 50, 10, and 1 grains	49°·92	57°·57	30·82
,, 4 . . .	Gilt lb. 32 + 4 oz. + 500, 300, 50, 10, and 0·3 grains	62·75	57·62	30·75
,, 5 . . .	Gilt lb. 32 + 4 oz. + 500, 300, 50, 10, 1, ·5, 0·2, 0·1 grains	53·03	57·60	30·55

Then at 62° Fahr., B = 30 inches,

$$\text{Log vol. Q in grains} = 3·7643360, \text{ or}$$
$$v_{Q} = 5812·14 \text{ grains.}$$

The density of the quartz cylinder being :—

At 62°.
2·265425

Therefore, as the cubic contents of Q are equal to 23·04015 cubic inches, the mass of one cubic inch at 62° Fahr. (B. 30 inches) may, by these experiments with Q, be taken as 252·261 grains.

DIAMETER of Sphere S.

Date. 1888.	*t.* Fahr.	Diameter of S at eight different positions, as compared with a standard distance between two contact points, P–P′.								Mean diameter.	Standard distance between contact points P—P′.
		1.	2.	3.	4.	5.	6.	7.	8.		
	°	inches.	inches.	inches.	inches.	inches.	inches.	inches.	inches.	inches.	inches.
Nov. 15 . . .	62·2	0·05013	0·05024	0·05025	0·05025	0·00026	0·05026	0·05023	0·05025	0·050234	0·05796
,, 16 . . .	64·0	0·05013	0·05014	0·05013	0·05023	0·05026	0·05027	0·05024	0·05027	0·050209	0·05796
,, 17 . . .	63·2	0·05014	0·05018	0·05025	0·05017	0·05025	0·05026	0·05027	0·05026	0·050225	0·05795
,, 19 . . .	63·3	0·05014	0·05018	0·05013	0·05018	0·05025	0·05026	0·05027	0·05026	0·050209	0·05795
,, 21 . . .	62·2	0·05013	0·05024	0·05013	0·05025	0·05026	0·05027	0·05025	0·05026	0·050224	0·05796
,, 23 . . .	63·0	0·05013	0·05024	0·05013	0·05024	0·05026	0·05027	0·05027	0·05026	0·050225	0·05796
Mean . .	62·98	0·05013	0·05020	0·05017	0·05022	0·050257	0·050265	0·050255	0·05026	0·050221	0·057957

The measurements were made by means of a pair of micrometer microscopes. It was found that the sphere was 0·00774 inch shorter in diameter than the steel 6-inch bar (*t.* 62°·98 Fahr.).

After allowing for the difference between the rates of expansion of the steel standard 6-inch and the brass sphere, and the error of the steel standard in relation to the imperial yard, the diameter of the sphere is 5·992444 inches. On the sphere, however, two lines had been traced, for which a correction is to be made. These lines are triangular in section, being nearly 0·01 inch wide and 0·005 inch deep, therefore the actual contents of S would be at 62° Fahr. = 112·6694096 cubic inches.

WEIGHING of Sphere S in Air.

Date, 1888.	Description of weights used. (*w*.) $\Delta w = 8 \cdot 143$.	Result.	*t*. Fahr.	B. inches.
	Before immersion.			
Dec. 10 . . .	2 lb. + gilt lbs. Nos. 31 + 32 ; + gilt grains 300, 100, 10 .	S = 28410·113 grains . .	61°·65	in. 30·17
„ 11 . . .	2 lb. + gilt lbs. Nos. 31 + 32 ; + gilt grains 300, 100, 10 .	S = 28409·813 „ . .	57·70	30·26
„ 12 . . .	2 lb. + gilt lbs. Nos. 31 + 32 ; + gilt grains 300, 100, 10 .	S = 28409·802 „ . .	60·28	30·37
„ 17 . . .	2 lb. + gilt lbs. Nos. 31 + 32 ; + gilt grains 300, 100, 10 .	S = 28409·917 „ . .	58·32	30·35
	After immersion.			
„ 18 . . .	2 lb. + gilt lbs. Nos. 31 + 32 ; + gilt grains 300, 100, 10 .	S = 28409·918 „ . .	58·10	30·18

The following are the weights of air displaced by S and *w* respectively :—

	S displaces	*w* displaces
Dec. 10	34·810 grains.	4·198 grains.
„ 11	35·249 „	4·245 „
„ 12	35·200 „	4·239 „
„ 17	35·193 „	4·238 „
„ 18	35·129 „	4·230 „

In normal air S displaces 34·643 and *w* 4·171 grains respectively; whence *in vacuo*, and in normal air, the weights in grains of the sphere are as follows :—

In vacuo.		In normal air.
28,440·725 grains.	. . . .	28,410·253 grains.
28,440·817 ,,	. . . .	28,410·345 ,,
28,440·763 ,,	. . . .	28,410·291 ,,
28,440·772 ,,	. . . .	28,410·300 ,,
28,440·810 ,,	. . . .	28,410·345 ,,
28,440·777 grains.		28,410·307 grains.

Or the logarithms of the weights in grains are :—

In vacuo.		In air.
4·4539414	. . . .	4·4534759

Weighing of S in Water.

Date, 1888.	Difference of equipoise w'.	Mean t. water.	Mean t. air.	B.
	gr.	°	°	inches.
Dec. 12	Gilt grains weights added, 27·02	54·58	60·22	30·37
,, 12	,, ,, ,, 27·01	54·72	60·50	30·37
,, 13	,, ,, ,, 24·70	56·63	59·20	30·30
,, 17	,, ,, ,, 23·72	57·60	58·48	30·35
,, 18	,, ,, ,, 23·65	57·48	58·00	30·18

S just floats in water and was suspended and kept in position by a "sinker." The weights of water displaced by the sinker, with and without the gravimeter attached to the sinker, were ascertained by separate equipoises, the small grain weights added each day representing the differences of such equipoises.

The density of the weights w' used was 8·143 ; hence the density of S is—

$$\begin{aligned}\Delta_S &= 0·999333\\ &= 0·999331\\ &= 0·999302\\ &= 0·999177\\ &= 0·999181\\ & \overline{0·999265}\end{aligned}$$

or log $\Delta_S = \bar{1}·9996553$.

As the weight of S in air is 28410·307 grains, the volume of S in grains at 62° Fahr. will be 28426·61 grains. As the cubic contents of S have been taken as 112·6694096 cubic inches, the mass of one cubic inch of water ($t. = 62°$) would be 252·301 grains.

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Final Result.

The measurements of the sphere S appear to afford more accurate results than the measurements of the cylinders. The measurements of the larger cylinder C afford a more accurate result than those of the cylinder Q; although, on the other hand, there appears to be less uncertainty as to the volume of the solid quartz cylinder at the various temperatures.

If all the weighings and measurements of the three gravimeters had been of equal value, then a mean (252·276) of the three results :—

C. 252·267 grains
S. 252·301 ,,
Q. 252·261 ,,

might have been taken as the final result κ of the present experiments; but, for the considerations above stated, it was thought desirable to take as the final result $\kappa = C + 3S + Q/5$, or 252·286 grains $\pm$ 0·0002 grain, as the mass of the cubic inch of distilled water freed from air, weighed in air at the temperature of 62° Fahr., the barometer being reduced to 30 inches, against brass weights of the density of 8·143.

A cubic foot of such water under the above conditions would weigh 435950·208 grains, or 62·278601 lbs. avoirdupois.

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